Exponential Stabilization for Neutral Neural Networks with Time-varying Delays by Periodically Intermittent Control

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Abstract: This paper focuses on the problem of exponential stabilization analysis for a class of neutral neural networks with multiple time-varying delays by periodically intermittent control. We construct a suitable Lyapunov-Krasovskii functional and utilize Jensen's inequality to establish sufficient conditions that guarantee the exponential stabilization of neutral neural networks with mixed time-varying delays in terms of linear matrix inequalities. We then present the computing method for the gain matrix of periodically intermittent control. Finally, a numerical example illustrates the effectiveness of the obtained result.

Keywords: exponential stabilization; neutral neural networks; time-varying delays; linear matrix inequality (LMI); periodically intermittent control.

1. Introduction

During the past few decades, neural networks have been extensively investigated due to their wide applications in a variety of fields, such as power systems, secure communication, pattern recognition, as-sociative memory and so on [1-4]. Neutral neural network is a kind of neural networks that involves the derivative of the past state. It is widely used in some fields, such as lossless transmission line, mathematics, ecological system, controlled constrained manipulators and so on [5-7]. It is generally known that neutral neural network has more complicated characteristics. As a consequence, neutral neural network have attracted many scholar's attention, especially the studies of stability and synchronization [8-12]. In [8], the equilibrium and stability properties of a class of neutral-type neural network model with discrete time delays were studied. In [9], Mahmoud et al. investigated the problem of robust global exponential stability analysis for a class of neutral-type neural networks. Global exponential stability condition was derived by employing new Lyapunov-Krasovskii functional and the integral inequality.

Intermittent control has aroused a wide range of interest because of its extensive applications [1]. When the system output is measured intermittently rather than continuously, this control is a kind of effective approach. Intermittent control have been extensively studied and rapidly developed in the fields of chaotic systems and neural networks [1, 13-18]. In [13], Zhang et al. investigated the exponential stabilization for neutral-type neural networks with mixed interval time-varying delays by using intermittent control. In [14], Huang et al. studied stabilization of delayed chaotic neural networks by periodically intermittent control. In [16], the exponential stabilization and synchronization of neural networks with time-varying delays was considered via periodically intermittent control.

In reality, time delays are unavoidably due to the finite signal propagation time in the biological networks and the finite switching speeds of the amplifiers [3]. They are encountered in various systems, such as rolling mill, microware oscillator, chemical process, hydraulic systems and so on [17,18]. The existence of time delay may lead to instability and oscillation such that the performances of neural networks are degraded. Therefore, it is clear that in both theory and practice the stability analysis of delayed neural networks is of great importance. Up to our knowledge, there have been few results of an investigation for the stabilization of neural networks of neutral type with time-varying delay via intermittent control, which remains as an interesting research topic.

Motivated by the above statements, this paper considers the problem of exponential stabilization for neutral neural networks with time-varying delay via periodically intermittent control. By employing new Lyapunov-Krasovskii functional and the integral inequality, new sufficient conditions for exponential stabilization of neutral neural networks are derived based on the intermittent control. The developed stabilization criteria are delay-dependent and characterized by linear-matrix inequalities (LMIs). The developed results are less conservative than previous published ones in the literature, which are illustrated by representative numerical example.

This paper is organized as follows. In Section 2, the problem description and preliminaries are stated and some lemmas and a definition are given. In Section 3, some new criteria are established to guarantee the exponential stabilization of neutral neural networks. In Section 4, a numerical example is given to illustrate the effectiveness of the results obtained. Some conclusions are drawn in Section 5.

Notations: Throughout this paper, R^n denotes the n-dimensional Euclidean space. $R^{n\times m}$ is the set of all $n\times m$ real matrices; $*$ represents the elements below the main diagonal of a symmetric matrix. M^T means the transpose of M; $\|\cdot\|$ is the Euclidean norm of a vector; $M>0(<0,\le 0,\ge 0)$ means that the matrix is symmetric positive (negative, semi-negative, semi-positive) definite matrix. I is an appropriately dimensioned identify matrix. $\lambda_{\max}(M)$ and $\lambda_{\min}(M)$ stand for the minimum and maximal eigenvalue of a matrix M respectively. $\underset{x\in[a,b]}{Sup}\, f(x)$ denotes the minimum value of upper bounds of the function $f(x)$ on the interval $[a,b]$.

2. Problem statement and preliminaries

In this section, a class of neutral neural networks with mixed time-varying delays is considered, and its model is represented as follows

$$\begin{aligned}&\dot{x}(t)-C\dot{x}(t-\tau(t))=-Ax(t)+Bf(x(t))+Df(x(t-h(t)))+Eu(t), \quad t\ge 0,\\&x(t)=\phi(t), \quad \forall t\in[-\bar{h},0],\end{aligned} \tag{1}$$

where $x(t)=[x_1(t),x_2(t),\cdots,x_n(t)]^T\in R^n$ is the state vector of the neural networks associated with n neurons at time t, $f(t)=[f_1(t),f_2(t),\cdots,f_n(t)]^T$ denotes the neuron activation function with $f(0)=0$, $u(t)\in R^n$ is the control input vector; the matrices $A=diag\{a_1,a_2,...,a_n\}$ is a diagonal matrix with positive entries $a_i>0$, $i=1,2,...,n$, $C>0$ is a known constant matrix with appropriate dimension. B and D are the connection weight matrix and delayed connection weight matrix respectively. E is a reversible matrix. The initial condition $\phi(t)$ denotes a continuous vector-valued initial function on the interval $[-\bar{h},0]$.

Assumption 1. The time delays $h(t)$ and $\tau(t)$ are time-varying differentiable functions satisfying

$$\begin{aligned}&0\le h(t)\le h, \quad \dot{h}(t)\le h_d,\\&0\le \tau(t)\le \tau, \quad \dot{\tau}(t)\le \tau_d, \quad \bar{h}=\max\{h,\tau\},\end{aligned} \tag{2}$$

where h and τ are the upper bound of $h(t)$ and $\tau(t)$ respectively. h_d and τ_d are the real constants.

Assumption 2. [2] The nonlinear activation function $f(\cdot)$ satisfies the following condition, for any $i=1,2,\cdots,n,$ there exist constants l_i^- and l_i^+ such that

$$l_i^-\le \frac{f_i(x)-f_i(y)}{x-y}\le l_i^+, \quad i=1,2,\cdots,n, \tag{3}$$

where $x,y\in\mathbb{R}$, $x\ne y$.

For expression convenience, we denote

$$\begin{aligned}&L_1=diag\{l_1^-l_1^+,l_2^-l_2^+,\cdots,l_n^-l_n^+\},\\&L_2=diag\{\frac{l_1^-+l_1^+}{2},\frac{l_2^-+l_2^+}{2},\cdots,\frac{l_n^-+l_n^+}{2}\},\\&L_3=diag\{\max\{|l_1^-|,|l_1^+|\},\max\{|l_2^-|,|l_2^+|\},\cdots,\max\{|l_n^-|,|l_n^+|\}\},\end{aligned}$$

For system (1) with initial value, we consider an intermittent state feedback controller expressed as follows:

$$u(t)=\begin{cases}Kx(t), & t\in[kT,kT+\delta),\\0, & t\in[kT+\delta,(k+1)T),\end{cases} \tag{4}$$

for any nonnegative integer k, where K is a constant control gain, T is the control period, $0<\delta\le T$, and δ is the so-called control width.

When the intermittent state-feedback control (4) is applied to (1), system (1) can be rewritten as follows:

$$\begin{cases}\dot{x}(t)-C\dot{x}(t-\tau(t))=-(A-EK)x(t)+Bf(x(t))+Df(x(t-h(t))), & t\in[kT,kT+\delta),\\\dot{x}(t)-C\dot{x}(t-\tau(t))=-Ax(t)+Bf(x(t))+Df(x(t-h(t))), & t\in[kT+\delta,(k+1)T),\\x(t)=\phi(t), \quad \forall t\in[-\bar{h},0].\end{cases} \tag{5}$$

Definition 1. System (1) is said to be α- exponentially stabilizable via intermittent state feedback control (4), if there exist $\alpha>0$ and $N>0$ such that the solution $x(t)$ of system (5) satisfies

$$\|x(t)\| \le Ne^{-\alpha t}\|\phi\|, \quad \forall t \ge 0,$$

where $\|\phi\| = \sup_{\theta\in[-\bar{h},0]} \|x(\theta)\|$.

In order to proof the main results, we introduce the following lemma firstly.

Lemma 1. [19] (Jensen's Inequality) For any matrix $R \in \mathbb{R}^{n\times n}$, $R = R^T > 0$, scalars α and $\beta : \beta < \alpha$, vector $x : [\beta, \alpha] \to \mathbb{R}^n$ such that the integration concerned are well defined, then:

$$-\int_\beta^\alpha x^T(s)Rx(s)ds \le -\frac{1}{\alpha-\beta}\left(\int_\beta^\alpha x(s)ds\right)^T R\left(\int_\beta^\alpha x(s)ds\right),$$

$$-\int_\beta^\alpha\int_\theta^\alpha x^T(s)Rx(s)dsd\theta \le -\frac{2}{(\alpha-\beta)^2}\left(\int_\beta^\alpha\int_\theta^\alpha x(s)Rx(s)dsd\theta\right)^T R\left(\int_\beta^\alpha\int_\theta^\alpha x(s)Rx(s)dsd\theta\right).$$

3. Main results

In this section, we address the problems on exponential stabilization analysis by means of Lyapunov-Krasovskii functional method. The main result is stated as follows.

Theorem 1. Suppose that Assumption 1 and Assumption 2 are satisfied. For given constants $\alpha > 0$ and γ, the system (1) is α- exponentially stabilizable via intermittent state-feedback controller (4), if there exist matrices $P > 0$, $Q_i > 0\,(i = 1,2,3,4)$, $R_i > 0\,(i = 1,2)$, $W_1 > 0$, $W_2 > 0$, Z such that

$$\Pi = \begin{bmatrix} \Pi_{11} & 0 & 0 & 0 & 0 & 0 & 0 & L_2W_1+PB & PD & \Pi_{1,10} & \Pi_{1,11} \\ * & \Pi_{22} & 0 & 0 & 0 & 0 & 0 & 0 & L_2W_2 & 0 & 0 \\ * & * & \Pi_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Pi_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Pi_{55} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Pi_{66} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Pi_{77} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -W_1 & 0 & (PB)^T & -(PB)^T \\ * & * & * & * & * & * & * & * & -W_2 & (PD)^T & -(PD)^T \\ * & * & * & * & * & * & * & * & * & -2P & PC+P \\ * & * & * & * & * & * & * & * & * & * & -2PC \end{bmatrix} < 0, \tag{6}$$

$$\hat{\Pi} = \begin{bmatrix} \hat{\Pi}_{11} & 0 & 0 & 0 & 0 & 0 & 0 & L_2W_1+PB & PD & \hat{\Pi}_{1,10} & \hat{\Pi}_{1,11} \\ * & \Pi_{22} & 0 & 0 & 0 & 0 & 0 & 0 & L_2W_2 & 0 & 0 \\ * & * & \Pi_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Pi_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Pi_{55} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Pi_{66} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Pi_{77} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -W_1 & 0 & (PB)^T & -(PB)^T \\ * & * & * & * & * & * & * & * & -W_2 & (PD)^T & -(PD)^T \\ * & * & * & * & * & * & * & * & * & -2P & PC+P \\ * & * & * & * & * & * & * & * & * & * & -2PC \end{bmatrix} < 0, \tag{7}$$

$$\alpha\delta - \rho(T-\delta) > 0, \tag{8}$$

where

$$\Pi_{11} = 2\alpha P + Q_1 + Q_2 + Q_3 + Q_4 + hR_1 + \tau R_2 - L_1W_1 - 2PA + 2Z,$$
$$\hat{\Pi}_{11} = 2\alpha P + Q_1 + Q_2 + Q_3 + Q_4 + hR_1 + \tau R_2 - L_1W_1 - 2PA,$$
$$\Pi_{1,10} = (-PA+Z)^T,$$
$$\hat{\Pi}_{1,10} = -(PA)^T,$$

$$\Pi_{1,11} = (PA - Z)^T + PC,$$
$$\hat{\Pi}_{1,11} = (PA)^T + PC,$$
$$\Pi_{22} = -(1 - h_d)e^{-2\alpha h}Q_2 - L_1W_2,$$
$$\Pi_{33} = -e^{-2\alpha h}Q_1,$$
$$\Pi_{44} = -(1 - \tau_d)e^{-2\alpha\tau}Q_4,$$
$$\Pi_{55} = -e^{-2\alpha\tau}Q_3,$$
$$\Pi_{66} = -he^{-2\alpha h}R_1,$$
$$\Pi_{77} = -\tau e^{-2\alpha\tau}R_2,$$
$$\rho = \gamma - \alpha.$$

Moreover, the gain matrix in the periodically intermittent controller (4) is given by $K = (PE)^{-1}Z$.

Proof. Choose a Lyapunov-Krasovskii functional candidate as:

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{9}$$

where

$$\begin{aligned} V_1(t) &= x^T(t)Px(t), \\ V_2(t) &= \int_{t-h}^{t} e^{2\alpha(s-t)}x^T(s)Q_1x(s)ds + \int_{t-h(t)}^{t} e^{2\alpha(s-t)}x^T(s)Q_2x(s)ds \\ &\quad + \int_{t-\tau}^{t} e^{2\alpha(s-t)}x^T(s)Q_3x(s)ds + \int_{t-\tau(t)}^{t} e^{2\alpha(s-t)}x^T(s)Q_4x(s)ds, \\ V_3(t) &= \int_{-h}^{0}\int_{t+\theta}^{t} e^{2\alpha(s-t)}x^T(s)R_1x(s)dsd\theta + \int_{-\tau}^{0}\int_{t+\theta}^{t} e^{2\alpha(s-t)}x^T(s)R_2x(s)dsd\theta. \end{aligned}$$

It is clear that

$$V(t) \geq \lambda_{\min}(P)\|x(t)\|^2 . \tag{10}$$

Calculating the time derivatives of V_i, $i = 1, 2, 3,$ along the trajectory of system (1) yields

$$\begin{aligned} \dot{V}(t) + 2\alpha V(t) &\leq 2x^T(t)P\dot{x}(t) + 2\alpha x^T(t)Px(t) + x^T(t)Q_1x(t) - e^{-2\alpha h}x^T(t-h)Q_1x(t-h) \\ &\quad + x^T(t)Q_2x(t) - e^{-2\alpha h(t)}x^T(t-h(t))Q_2x(t-h(t))(1-\dot{h}(t)) + x^T(t)Q_3x(t) \\ &\quad - e^{-2\alpha\tau}x^T(t-\tau)Q_3x(t-\tau) + x^T(t)Q_4x(t) + hx^T(t)R_1x(t) \\ &\quad - e^{-2\alpha\tau(t)}x^T(t-\tau(t))Q_4x(t-\tau(t))(1-\dot{\tau}(t)) - \int_{t-h}^{t} e^{2\alpha(s-t)}x^T(s)R_1x(s)ds \\ &\quad + \tau x^T(t)R_2x(t) - \int_{t-\tau}^{t} e^{2\alpha(s-t)}x^T(s)R_2x(s)ds \\ &\leq 2x^T(t)P\dot{x}(t) + 2\alpha x^T(t)Px(t) + x^T(t)Q_1x(t) - e^{-2\alpha h}x^T(t-h)Q_1x(t-h) \\ &\quad + x^T(t)Q_2x(t) - e^{-2\alpha h(t)}x^T(t-h(t))Q_2x(t-h(t))(1-h_d) + x^T(t)Q_3x(t) \\ &\quad - e^{-2\alpha\tau}x^T(t-\tau)Q_3x(t-\tau) + x^T(t)Q_4x(t) + hx^T(t)R_1x(t) \\ &\quad - e^{-2\alpha\tau(t)}x^T(t-\tau(t))Q_4x(t-\tau(t))(1-\tau_d) - \int_{t-h}^{t} e^{2\alpha(s-t)}x^T(s)R_1x(s)ds \\ &\quad + \tau x^T(t)R_2x(t) - \int_{t-\tau}^{t} e^{2\alpha(s-t)}x^T(s)R_2x(s)ds. \end{aligned} \tag{11}$$

By using Lemma 1, it can be seen that:

$$\begin{aligned} -\int_{t-h}^{t} e^{2\alpha(s-t)}x^T(s)R_1x(s)ds &\leq -e^{-2\alpha h}\int_{t-h}^{t} x^T(s)R_1x(s)ds \\ &\leq -e^{-2\alpha h}\frac{1}{h}\left(\int_{t-h}^{t} x(s)ds\right)^T R_1\left(\int_{t-h}^{t} x(s)ds\right), \end{aligned} \tag{12}$$

$$\begin{aligned} -\int_{t-\tau}^{t} e^{2\alpha(s-t)}x^T(s)R_2x(s)ds &\leq -e^{-2\alpha\tau}\int_{t-\tau}^{t} x^T(s)R_2x(s)ds \\ &\leq -e^{-2\alpha\tau}\frac{1}{\tau}\left(\int_{t-\tau}^{t} x(s)ds\right)^T R_2\left(\int_{t-\tau}^{t} x(s)ds\right). \end{aligned} \tag{13}$$

Furthermore, for any matrices $W_1 > 0$ and $W_2 > 0$, utilizing Assumption 2, we have

$$\begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^T \begin{bmatrix} -L_1W_1 & L_2W_1 \\ * & -W_1 \end{bmatrix} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} \geq 0, \tag{14}$$

$$\begin{bmatrix} x(t-h(t)) \\ f(x(t-h(t))) \end{bmatrix}^T \begin{bmatrix} -L_1W_2 & L_2W_2 \\ * & -W_2 \end{bmatrix} \begin{bmatrix} x(t-h(t)) \\ f(x(t-h(t))) \end{bmatrix} \geq 0. \tag{15}$$

In the following, we consider two cases in calculating the derivative of Lyapunov-Krasovskii functional: $t \in [kT, kT+\delta)$ and $t \in [kT+\delta, (k+1)T)$.

Case 1: For $t \in [kT, kT+\delta)$. The first subsystem of (5) can be written as

$$\dot{x}(t) = C\dot{x}(t-\tau(t)) - (A-EK)x(t) + Bf(x(t)) + Df(x(t-h(t))), \quad t \in [kT, kT+\delta),$$

It is easy to see that

$$2\left[\dot{x}^T(t) - \dot{x}^T(t-\tau(t)) + x^T(t)\right]P\left[C\dot{x}(t-\tau(t)) - (A-EK)x(t) + Bf(x(t)) + Df(x(t-h(t))) - \dot{x}(t)\right] = 0. \tag{16}$$

Setting

$$Z = PEK, \tag{17}$$

and combining (11)-(16), we get that

$$\dot{V}(t) + 2\alpha V(t) \leq \xi^T(t) \prod \xi(t), \tag{18}$$

where

$$\xi(t) = \left[\xi_1^T(t) \quad \xi_2^T(t) \quad \xi_3^T(t)\right]^T,$$

$$\xi_1(t) = \left[x^T(t) \quad x^T(t-h(t)) \quad x^T(t-h) \quad x^T(t-\tau(t)) \quad x^T(t-\tau)\right]^T,$$

$$\xi_2(t) = \left[\frac{1}{h}(\int_{t-h}^{t} x(s)ds)^T \quad \frac{1}{\tau}(\int_{t-\tau}^{t} x(s)ds)^T\right]^T,$$

$$\xi_3(t) = \left[f^T(x(t)) \quad f^T(x(t-h(t))) \quad \dot{x}(t) \quad \dot{x}(t-\tau(t))\right]^T,$$

and $\prod$ is given by (6).

As a result, since Eq. (6) holds, it is deduced from Eq. (18) that

$$\dot{V}(t) + 2\alpha V(t) \leq 0. \tag{19}$$

Now, in order to facilitate the estimation of the bound of $V(t)$, let us define $v_1(t) = e^{2\alpha t}V(t)$. Furthermore, it is clear that $v_1(t)$ is a monotonically decreasing function on $t \in [kT, kT+\delta)$. We have

$$v_1(t) \leq v_1(kT), \tag{20}$$

$$v_1(kT+\delta) \leq v_1(kT). \tag{21}$$

Then,

$$V(t) \leq V(kT)e^{-2\alpha(t-kT)}, \tag{22}$$

$$V(kT+\delta) \leq V(kT)e^{-2\alpha\delta}. \tag{23}$$

Case 2: For $t \in [kT+\delta, (k+1)T)$. The second subsystem of (5) can be written as

$$\dot{x}(t) = C\dot{x}(t-\tau(t)) - Ax(t) + Bf(x(t)) + Df(x(t-h(t))), \quad t \in [kT+\delta, (k+1)T).$$

It is easy to see that

$$2\left[\dot{x}^T(t) - \dot{x}^T(t-\tau(t)) + x^T(t)\right]P\left[C\dot{x}(t-\tau(t)) - Ax(t) + Bf(x(t)) + Df(x(t-h(t))) - \dot{x}(t)\right] = 0. \tag{24}$$

From (11)-(15) and (24), we get that

$$\begin{aligned} \dot{V}(t) + 2\alpha V(t) &\leq \xi^T(t)\hat{\prod}\xi(t) + 2\gamma x^T(t)Px(t) \\ &\leq \xi^T(t)\hat{\prod}\xi(t) + 2\gamma V(t), \end{aligned} \tag{25}$$

where $\hat{\prod}$ is given by (7). By using the condition (7), we have

$$\dot{V}(t) - 2(\gamma-\alpha)V(t) = \dot{V}(t) - 2\rho V(t) \leq 0. \tag{26}$$

As well, let us define $v_2(t) = e^{-2\rho t}V(t)$. Furthermore, it is easy to see that $v_2(t)$ is a monotonically decreasing function on $t \in [kT+\delta, (k+1)T)$. We have

$$v_2(t) \leq v_2(kT+\delta), \tag{27}$$

$$v_2((k+1)T) \leq v_2(kT+\delta). \tag{28}$$

Then,

$$V(t) \leq V(kT+\delta)e^{2\rho(t-kT-\delta)}, \tag{29}$$

$$V((k+1)T) \leq V(kT+\delta)e^{2\rho(T-\delta)}. \tag{30}$$

From (23) and (30), we have

$$V((k+1)T) \le V(0)e^{-(k+1)[2\alpha\delta-2\rho(T-\delta)]}. \tag{31}$$

$$V(kT+\delta) \le V(0)e^{-2\alpha\delta(k+1)+2\rho(T-\delta)k}. \tag{32}$$

Therefore, on one hand, for $t \in [kT, kT+\delta)$, from (22), (31) and (8), we get

$$\begin{aligned} V(t) &\le V(kT)e^{-2\alpha(t-kT)} \\ &\le V(0)e^{-k[2\alpha\delta-2\rho(T-\delta)]}e^{-2\alpha(t-kT)} \\ &\le V(0)e^{-k[2\alpha\delta-2\rho(T-\delta)]} \\ &= V(0)e^{2\alpha\delta-2\rho(T-\delta)}e^{-(2\alpha\delta-2\rho(T-\delta))[(kT+\delta)+(T-\delta)]/T} \\ &\le \beta_1 V(0)e^{\frac{-(2\alpha\delta-2\rho(T-\delta))t}{T}}, \end{aligned} \tag{33}$$

where

$$\beta_1 = e^{\frac{2\alpha\delta-2\rho(T-\delta))}{T}\delta}.$$

On the other hand, for $t \in [kT+\delta, (k+1)T)$, from (29), (32) and (8), we get

$$\begin{aligned} V(t) &\le V(kT+\delta)e^{2\rho(t-kT-\delta)} \\ &\le V(0)e^{-2\alpha\delta(k+1)+2\rho(T-\delta)k}e^{2\rho(t-kT-\delta)} \\ &\le V(0)e^{-2\alpha\delta(k+1)+2\rho(T-\delta)k}e^{2|\rho|[(k+1)T-kT-\delta)]} \\ &= V(0)e^{\frac{-(2\alpha\delta-2\rho(T-\delta))(k+1)T}{T}}e^{2(|\rho|-\rho)(T-\delta)} \\ &\le \beta_2 V(0)e^{\frac{-(2\alpha\delta-2\rho(T-\delta))t}{T}}, \end{aligned} \tag{34}$$

where

$$\beta_2 = e^{2(|\rho|-\rho)(T-\delta)}.$$

Let $\beta = \{\beta_1, \beta_2\}$. From (33) and (34), we have

$$V(t) \le \beta V(0)e^{\frac{-(2\alpha\delta-2\rho(T-\delta))t}{T}}, \quad \forall t \ge 0. \tag{35}$$

Obviously

$$\begin{aligned} V(0) &= V_1(0)+V_2(0)+V_3(0) \\ &= x^T(0)Px(0) + \int_{-h}^{0} e^{2\alpha s}x^T(s)Q_1x(s)ds + \int_{-h(0)}^{0} e^{2\alpha s}x^T(s)Q_2x(s)ds \\ &\quad + \int_{-\tau}^{0} e^{2\alpha s}x^T(s)Q_3x(s)ds + \int_{-\tau(0)}^{0} e^{2\alpha s}x^T(s)Q_4x(s)ds \\ &\quad + \int_{-h}^{0}\int_{\theta}^{0} e^{2\alpha s}x^T(s)R_1x(s)dsd\theta + \int_{-\tau}^{0}\int_{\theta}^{0} e^{2\alpha s}x^T(s)R_2x(s)dsd\theta \\ &\le N_1\|\phi\|^2, \end{aligned} \tag{36}$$

where

$$N_1 = \lambda_{\max}(P) + h\lambda_{\max}(Q_1) + h\lambda_{\max}(Q_2) + \tau\lambda_{\max}(Q_3) + \tau\lambda_{\max}(Q_4) + \frac{h^2}{2}\lambda_{\max}(R_1) + \frac{\tau^2}{2}\lambda_{\max}(R_2).$$

Hence, from (10), (35) and (36), we get

$$\|x(t)\| \le \sqrt{\frac{\beta N_1}{\lambda_{\min}(P)}}e^{\frac{-(\alpha\delta-\rho(T-\delta))t}{T}}\|\phi\|, \quad \forall t \ge 0. \tag{37}$$

As a result, according to Definition 1 and (37), the neutral neural networks (1) with multiple time-varying delays are α- exponentially stabilizable under the intermittent controller (4). Furthermore, the state-feedback intermittent gain matrix $K = (PE)^{-1}Z$. This ends the proof.

Remark 1. When $\tau(t) = h(t)$, the system (1) can be written as

$$\begin{aligned} &\dot{x}(t) - C\dot{x}(t-h(t)) = -Ax(t) + Bf(x(t)) + Df(x(t-h(t))) + Eu(t), \quad t \ge 0, \\ &x(t) = \phi(t), \quad \forall t \in [-\bar{h}, 0], \end{aligned} \tag{38}$$

then, we have the following corollary.

Corollary 1. Suppose that Assumption 2 is satisfied. For given constants $\alpha > 0$ and γ, the system (38) is α-exponentially stabilizable via intermittent state-feedback controller (4), if there exist matrices $P > 0$, $Q_i > 0 (i = 1,2)$, $R_1 > 0$, $W_1 > 0$, $W_2 > 0$, Z such that

$$\Pi = \begin{bmatrix} \Pi_{11} & 0 & 0 & 0 & L_2W_1 + PB & PD & \Pi_{17} & \Pi_{18} \\ * & \Pi_{22} & 0 & 0 & 0 & L_2W_2 & 0 & 0 \\ * & * & \Pi_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Pi_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & -W_1 & 0 & (PB)^T & -(PB)^T \\ * & * & * & * & * & -W_2 & (PD)^T & -(PD)^T \\ * & * & * & * & * & * & -2P & PC + P \\ * & * & * & * & * & * & * & -2PC \end{bmatrix} < 0, \tag{39}$$

$$\hat{\Pi} = \begin{bmatrix} \hat{\Pi}_{11} & 0 & 0 & 0 & L_2W_1 + PB & PD & \hat{\Pi}_{17} & \hat{\Pi}_{18} \\ * & \Pi_{22} & 0 & 0 & 0 & L_2W_2 & 0 & 0 \\ * & * & \Pi_{33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Pi_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & -W_1 & 0 & (PB)^T & -(PB)^T \\ * & * & * & * & * & -W_2 & (PD)^T & -(PD)^T \\ * & * & * & * & * & * & -2P & PC + P \\ * & * & * & * & * & * & * & -2PC \end{bmatrix} < 0, \tag{40}$$

$$\alpha\delta - \rho(T - \delta) > 0, \tag{41}$$

where

$$\begin{aligned} &\Pi_{11} = 2\alpha P + Q_1 + Q_2 + hR_1 - L_1W_1 - 2PA + 2Z, \\ &\hat{\Pi}_{11} = 2\alpha P + Q_1 + Q_2 + hR_1 - L_1W_1 - 2PA, \\ &\Pi_{17} = (-PA + Z)^T, \\ &\hat{\Pi}_{17} = -(PA)^T, \\ &\Pi_{18} = (PA - Z)^T + PC, \\ &\hat{\Pi}_{18} = (PA)^T + PC, \\ &\Pi_{22} = -(1 - h_d)e^{-2\alpha h}Q_2 - L_1W_2, \\ &\Pi_{33} = -e^{-2\alpha h}Q_1, \\ &\Pi_{44} = -he^{-2\alpha h}R_1, \\ &\rho = \gamma - \alpha. \end{aligned}$$

Moreover, the gain matrix in the periodically intermittent controller (4) is given by $K = (PE)^{-1}Z$.

Proof. Choose $Q_3 = 0$, $Q_4 = 0$, $R_2 = 0$ and

$$\begin{aligned} &\eta(t) = \begin{bmatrix} \eta_1^T(t) & \eta_2^T(t) & \eta_3^T(t) \end{bmatrix}^T, \\ &\eta_1(t) = \begin{bmatrix} x^T(t) & x^T(t - h(t)) & x^T(t - h) \end{bmatrix}^T, \\ &\eta_2(t) = \frac{1}{h}(\int_{t-h}^{t} x(s)ds), \\ &\eta_3(t) = \begin{bmatrix} f^T(x(t)) & f^T(x(t - h(t))) & \dot{x}^T(t) & \dot{x}^T(t - h(t)) \end{bmatrix}^T. \end{aligned}$$

The proof is similar to that for the Theorem 1 and thus it is omitted here.

Remark 2. When the time delays are not time-varying, the system (1) can be written as

$$\begin{aligned} &\dot{x}(t) - C\dot{x}(t - \tau) = -Ax(t) + Bf(x(t)) + Df(x(t - h)) + Eu(t), \quad t \ge 0, \\ &x(t) = \phi(t), \quad \forall t \in [-\bar{h}, 0], \end{aligned} \tag{42}$$

then, we have the following corollary.

Corollary 2. Suppose that Assumption 2. For given constants $\alpha > 0$ and γ, the system (42) is α- exponentially stabilizable via intermittent state-feedback controller (4), if there exist matrices $P > 0$, $Q_i > 0 (i = 1,2)$, $R_1 > 0$, $W_1 > 0$, $W_2 > 0$, Z such that

$$\Pi = \begin{bmatrix} \Pi_{11} & 0 & 0 & 0 & 0 & L_2W_1 + PB & PD & \Pi_{18} & \Pi_{19} \\ * & \Pi_{22} & 0 & 0 & 0 & 0 & L_2W_2 & 0 & 0 \\ * & * & \Pi_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Pi_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Pi_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -W_1 & 0 & (PB)^T & -(PB)^T \\ * & * & * & * & * & * & -W_2 & (PD)^T & -(PD)^T \\ * & * & * & * & * & * & * & -2P & PC + P \\ * & * & * & * & * & * & * & * & -2PC \end{bmatrix} < 0, \tag{43}$$

$$\hat{\Pi} = \begin{bmatrix} \hat{\Pi}_{11} & 0 & 0 & 0 & 0 & L_2W_1 + PB & PD & \hat{\Pi}_{18} & \hat{\Pi}_{19} \\ * & \Pi_{22} & 0 & 0 & 0 & 0 & L_2W_2 & 0 & 0 \\ * & * & \Pi_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Pi_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Pi_{55} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -W_1 & 0 & (PB)^T & -(PB)^T \\ * & * & * & * & * & * & -W_2 & (PD)^T & -(PD)^T \\ * & * & * & * & * & * & * & -2P & PC + P \\ * & * & * & * & * & * & * & * & -2PC \end{bmatrix} < 0, \tag{44}$$

$$\alpha\delta - \rho(T - \delta) > 0, \tag{45}$$

where

$$\Pi_{11} = 2\alpha P + Q_1 + Q_3 + hR_1 + \tau R_2 - L_1W_1 - 2PA + 2Z,$$
$$\hat{\Pi}_{11} = 2\alpha P + Q_1 + Q_3 + hR_1 + \tau R_2 - L_1W_1 - 2PA,$$
$$\Pi_{18} = (-PA + Z)^T,$$
$$\hat{\Pi}_{18} = -(PA)^T,$$
$$\Pi_{19} = (PA - Z)^T + PC,$$
$$\hat{\Pi}_{19} = (PA)^T + PC,$$
$$\Pi_{22} = -e^{-2\alpha h}Q_1 - L_1W_2,$$
$$\Pi_{33} = -e^{-2\alpha\tau}Q_3,$$
$$\Pi_{44} = -he^{-2\alpha h}R_1,$$
$$\Pi_{55} = -\tau e^{-2\alpha\tau}R_2,$$
$$\rho = \gamma - \alpha.$$

Moreover, the gain matrix in the periodically intermittent controller (4) is given by $K = (PE)^{-1}Z$.

Proof. Choose $Q_2 = 0$, $Q_4 = 0$ and

$$\zeta(t) = \begin{bmatrix} \zeta_1^T(t) & \zeta_2^T(t) & \zeta_3^T(t) \end{bmatrix}^T,$$
$$\zeta_1(t) = \begin{bmatrix} x^T(t) & x^T(t-h) & x^T(t-\tau) \end{bmatrix}^T,$$
$$\zeta_2(t) = \begin{bmatrix} \frac{1}{h}(\int_{t-h}^{t} x(s)ds)^T & \frac{1}{\tau}(\int_{t-\tau}^{t} x(s)ds)^T \end{bmatrix}^T,$$
$$\zeta_3(t) = \begin{bmatrix} f^T(x(t)) & f^T(x(t-h)) & \dot{x}(t) & \dot{x}(t-\tau) \end{bmatrix}^T.$$

The proof is similar to that for the Theorem 1 and thus it is omitted here.

4. Numerical example

In this section, we will provide a numerical example to illustrate the effectiveness and the merits of the obtained results.

Example 1. Consider the following neutral neural networks with time-varying delays

$$\dot{x}(t) - C\dot{x}(t-\tau(t)) = -Ax(t) + Bf(x(t)) + Df(x(t-h(t))) + Eu(t), \quad t \geq 0,$$

where

$$A = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}, \quad B = \begin{bmatrix} 6 & 15 \\ 1 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1.5 \\ 1.5 & 1 \end{bmatrix},$$

$$D = \begin{bmatrix} -4 & 2 \\ 2 & 4.8 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad f(x(t)) = \begin{bmatrix} \tanh(x_1(t)) \\ \tanh(x_2(t)) \end{bmatrix},$$

$$h(t) = 0.1\sin(t) + 0.2, \quad \tau(t) = 0.1\sin(t) + 0.1.$$

We have $l_1^- = l_1^- = 0,\ l_1^+ = l_1^+ = 1.$

For given $\alpha = 0.01, \gamma = 0.018,$ we take $h_d = 0.1, \tau_d = 0.1, h = 0.3, \tau = 0.2, T = 2, \delta = 1.9.$

By utilizing the MATLAB LMI Toolbox to solve the LMIs given in the Theorem 1, the feasible solutions can be obtained as follows:

$$P = \begin{bmatrix} 0.0133 & -0.0012 \\ -0.0012 & 0.0136 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 0.7840 & -0.0192 \\ -0.0192 & 0.7881 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 0.3590 & -0.0237 \\ -0.0237 & 0.3586 \end{bmatrix},$$

$$Q_3 = \begin{bmatrix} 0.7873 & -0.0192 \\ -0.0192 & 0.7915 \end{bmatrix}, \quad Q_4 = \begin{bmatrix} 0.5931 & -0.0227 \\ -0.0227 & 0.5974 \end{bmatrix}, \quad R_1 = \begin{bmatrix} 1.1929 & -0.0177 \\ -0.0177 & 1.1978 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} 1.1831 & -0.0135 \\ -0.0135 & 1.1867 \end{bmatrix}, \quad W_1 = \begin{bmatrix} 0.6296 & -0.0404 \\ -0.0404 & 0.6485 \end{bmatrix},$$

$$W_2 = \begin{bmatrix} 0.1709 & -0.0035 \\ -0.0035 & 0.1727 \end{bmatrix}, \quad Z = \begin{bmatrix} -0.0885 & -0.1118 \\ -0.1118 & -0.0637 \end{bmatrix}.$$

According to Theorem 1, the system is exponentially stabilization. Furthermore, a desired intermittent feedback gain matrix is derived by utilizing Theorem 1

$$K = \begin{bmatrix} -7.2112 & -8.8615 \\ -8.8615 & -5.4551 \end{bmatrix}.$$

5. Conclusion

This paper has addressed the problem of exponential stabilization for a class of neutral neural networks with multiple time-varying delays by periodically intermittent control. By constructing a suitable Lyapunov-Krasovskii functional and utilizing Jensen's inequality, the exponential stabilization criteria are presented in terms of linear matrix inequalities technique. Furthermore, the desired intermittent control gain matrix can be obtained by making use of the toolbox of MATLAB. Finally, a numerical example is given to illustrate the effectiveness of the obtained result.

6. References

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