Analysis on the Stability of Single Layer Spherical Lattice Shell

W. J. Su, X. Hai

College of Civil Engineering and Transportation, South China University of Technology, Guangzhou 510641, Guangdong, China
E-mail:wjsu@scut.edu.cn

Abstract: The problem of stability analysis is the most important one in the design of the spherical lattice shell (especially the single-layer lattice shell). Kiewitt lattice shell is one of many kinds that is widely employed in Engineering by us. Based on the stability theory of shell and the analysis using the general finite element software ANSYS, this paper focuses on the studying of how do the three core factors (Geometric nonlinearity, Material nonlinearity and Initial flaw) among many other factors influence the stability of a Kiewitt lattice shell.

Keywords: single layer lattice shell; stability; buckling analysis; geometric nonlinearity; material nonlinearity; initial flaw.

1. Introduction

In recent years, with the development of the world cup, Olympic Games and the World Expo, the large span space structure has been developed rapidly globally, the scale and size of large span and are also more and more complex. Many new materials and new technologies have been adopted in the world. Many magnificent and rich features of large space structure, has become a symbol of some cities or landmarks[1].

The birth and development of modern new technology comes in practical experience in the production and life of human, the invention of the Reinforced Concrete structure is a gardener to see the roots firmly bind soil inspiration, the birth of the Prestress technology is inspired from the ancient wood, stone tools and Thin Shell structure is derived from the observation and research of nature the seashell, egg shell and other. Reticulated shell structure (lattice shell) is a kind of space truss with similar structure, which is based on the bar, according to a certain rule and the space structure of the shell structure, containing both the nature of the bar and shell[2].

1.1 Characteristics of spherical lattice shell

Spherical lattice shell structure is a new type of structure, and has a broad development prospect. This is mainly because the shell structure has the following characteristics:

(1) Similar-sized grids

The lattice shell is a structure of the space bar, which is composed by similar-sized meshes or smaller units arranging along the surface of the shell.

(2) Uniform internal force distribution

There is no obvious primary and secondary relationship between the various members of the lattice shell structure. Each member can be almost balanced under various loads. Under the load, all bar members at any time bear the force they should to support the whole structure, and the bars of the shell structure mainly bear axial force. Thus its internal force distribution is more uniform and the peak stress is relatively small.

(3) Important node design

In the large span lattice shell structure, each node has a special importance. It is very difficult to transmit 3D force fluent. So the reasonability and reliability of the node structure is crucial for the simplicity of the shell’s construction and economical efficiency[3].

1.2 Types of single-layer spherical lattice shell

With the development of science and technology and people’s long-term unremitting efforts, lattice shell structure in the form of structure, materials and calculation methods have achieved great development. The structure of single layer lattice shell structure can be divided into the following types:

(1) Annularly ribbed spherical lattice shell

A rigid system is connected with the radial member of the ring. The radial force is subjected to the radial shaft. But the nodes need to bear the moment of the joints, so they are generally rigid connected. When the dome is relatively small, the supports bear large horizontal thrust, leading to a large amount of steel using for the rib. In order to overcome this shortcoming, weft-oriented purlin (solid web or grid structure) and ribs are connected into a rigid stereo system, called “annularly ribbed lattice shell”.

(2) Schwedler spherical lattice shell

Schwedler latticed shell is transformed and developed from annularly ribbed lattice shell. In addition to radial and weft-oriented bars outside, Schwedler spherical lattice shell also adds diagonal bars. Diagonal bar enhances the stiffness of the shell, and the ability to withstand large dissymmetrical load. Schwedler type spherical shell in
foreign countries, especially the United States, is still widely used, because of its rigidity, commonly used in large and medium span dome.

(3) Three-way spherical lattice shell
The grid is on the horizontal plane of the spherical surface. First of all the span is decided into n equal shares, then into normal triangular mesh, and finally the projects onto the sphere.

(4) Kiewitt lattice shell
Kiewitt lattice shell is consists of n (n is even) full-length radial bar system dividing sphere into n symmetric triangular meshes. And then within each of the triangle, the weft-oriented and diagonal bars to frame will be recomposed surface evenly divided into triangular meshes of similar size.

(5) Union type spherical shell
The grid of the union type spherical shell is made of two synclinal-crossed bars, with no radial members. The basic unit is diamond, and the grid partition method makes each node exchange cross bar number less and bar size in the whole structure as little as possible.

(6) Geodesic dome
"Geodesic" is the term in the earth Surveying Science, which means the shortest distance between any two points on the sphere. The grid division is more uniform, and the type of members and nodes is the least in all kinds of spherical shell, and most suitable for mass production in the factory.

Kiewitt shell contains the advantages of Schwedler lattice shell and Union type shell. Kiewitt shell mesh uniform, which contains relatively good mechanical properties, has higher bearing capacity for the seismic load. So it’s commonly used in large span structure. New Orleans stadium is a typical representative of the type of Kiewitt shell structure. This paper’s analysis is based on Kiewitt shell.

![Fig.1 Grid form of spherical lattice shell](image)

(a) Ribbed spherical lattice shell (b) Schwedler spherical lattice shell (c) Three-way spherical lattice shell (d) Kiewitt lattice shell (e) Union type spherical shell (f) Geodesic dome

1.3 Kewitte lattice shell
Kewitte lattice shell is composed of n (n=6, 8, 12......) Radial-length and radial oriented bars that dividing the spherical surface into n symmetrical fan-shaped areas, in accordance with the size of N, in the name of K8, K12 or K6...... And then by the weft direction and the diagonal bars, the surface is divided into triangular meshes, and the parallel and the right side are parallel. The number of planes and the number of mesh can be determined according to the diameter of the shell, the size of the roof panels and the load. This kind of lattice shell integrated the advantages of rotary division method and average triangulation method. Therefore, not only it has the similar size grids, but also uniform internal force distribution, commonly used in large, medium span of the dome.
2. Stability theory

In the internal force analysis of the structure, the main research contents are the balance of the structure, which is the study of the internal force distribution features and the mechanical properties of the structure under the balance state. It is often considered that the stability is a state of the structure, so the problem of structure’s stability often causes no special attention in engineering. But the design of the shell structure is usually affected by its stability and control\cite{4}. In 1963, the Bucharest Garrett, a span up to 93.5 meters of reticulated shell is overwhelmed by heavy snow. Through investigation and study, it was found that the tragedy is due to the instability caused by the collapse of the structure as a whole.

With the development of technology, the span of the shell structure is becoming larger, the thickness is thinner, the weight is lighter, the time is shorter, and the cost is lower. Of course, the seismic performance is good, and the rigidity is big\cite{5}. Large span, light weight and thin thickness have become the optimization results of the design of the structure. At the same time, the problem of the stability of the shell structure becomes more and more prominent. Stability is one of the most important problems in structural analysis, especially in the steel structure, the stability of the problem is often a factor affecting the safety of the structure.

2.1 Local instability and global instability

The failure of the lattice shell based on the geometrical appearance of the large deformation caused by the instability of the shell can be divided into local instability and global instability.
The global instability of a structure is a phenomenon that the structure with an external load, which has not reached the damage load, can’t be restored to the original equilibrium position (even removing the small load or interference) after being applied a very small load or geometric interference and causes a relatively large deviation, and even continue to deformation until it collapses. The stress of the structure is transferred from the film stress state to the bending stress state at the time of the shell’s failure.

Local instability of the structure is that the site of the deviation from the equilibrium position and the deformation of the site is confined to a certain range of the structure. That means part of the structure greatly deviate from the structural axis, but the structure’s overall geometric shape has not changed.

For the single layer lattice shell with small span and large rise-span ratio, the structure mainly resist external load with bending stress, and the failure mechanism is mainly strength failure, the local instability of the shell, which has a significant effect on the ultimate bearing capacity. For the large span, the relatively small rise-span ratio shell, the structure is mainly relying on its’ film stress resistance, and the main failure mechanism of this type is overall instability damage. This type of shell, the effect of local instability on the ultimate bearing capacity is relatively weak.

2.2 Instability factors
The instability problem of the shell has a certain complexity. The main factors that may cause the failure of the shell instability are: the structure of the initial defects, load type and distribution, structural support conditions, the curvature of the shell’s perimeter, geometric nonlinearity and material nonlinearity, bending stiffness and film stiffness of the shell structure. The initial defects and nonlinearity are the crucial factors that influence the possibility of structure’s failure.

The initial defects of the shell can be listed as follows: the geometric deviation of the structure, the structural initial bending and stress arising from the improper way of construction, the material defects, initial deflection of the load point, structural support’s deflection and so on. The study shows that little change of initial defects will lead to large variety of the critical load, and the decrease of the critical load is gradually weakened with the increase of the initial defects, whether it is the elastic or elastic-plastic shell.

The nonlinear structure of the structure can be divided into geometrical nonlinearity and material nonlinearity. The two party has a very important influence on the stability of the shell. The material properties directly affect the stiffness of the structure.

2.3 Assumption method of initial imperfection
With numerous joints and bars, requirement of the installation precision for the lattice shell is very high. However, due to inevitably error existing in the producing process of the nodes and bars, and installation deviation during the construction process like construction assembling, positioning line fixing, air lifting, pre-embedded parts, it causes initial force and displacement in the beginning before the structure actually work. These are called initial defects and they will weaken the structure’s bearing capacity and safety stock. In the calculation of the shell structure, how to consider the influence of initial defects, and how to assume the distribution of the initial imperfection is a big problem that the engineer needs to solve. There is a classical method in defect analysis: uniform defect mode method. In this paper, a uniform defect mode method is used to study the nonlinear problem of lattice shells.

Uniform defect mode method: the structure with N degrees of freedom has N order buckling modes with a certain kind of load, and the buckling mode is the trend of the structural displacement at the critical point. For the actual structure, there’s a trend along the modal deformation in the initial stage of loading. In theory, the lowest order buckling mode is the first and most likely buckling mode of the shell structure. The so-called uniform defect mode method considers as the most disadvantaged effect happens in the lowest order buckling mode, which is used to simulate the initial imperfection distribution of the structure. The perfect shell structure
introduces the initial imperfection of the lowest order buckling mode, which makes the shell more quickly in accordance with the lowest order buckling mode, and the buckling load is the smallest\textsuperscript{[15]}. Many references have been used to demonstrate the rationality of this method, and the results show that the defect distribution is close to the real situation.

3. Analysis of Kewitte latticed shell (using ANSYS)

The roof structure is a single-layer Kiewitt lattice shell structure, with 60m long span, rise span ratio of 1/6 (vector of 10m high). The structure is supported with surrounding restriction of X, Y, Z three-way rigid constraints, Q235B steel ring bars, and \( \varphi 203 \times 7 \) radial and diagonal bars. The weight of the roof is 0.35kN/m\(^2\), and live load is 0.50kN/m\(^2\) vertically. The general finite element software ANSYS is used for analysis, with the use of Beam4 element. Top view and side view of the shell are shown below.

![Fig.4 Top and side view of a Kewitte latticed shell (K6)](image)

### 3.1 Analysis of Nonlinear buckling

In this paper, a uniform defect mode method is used to study the nonlinear problem. The two cases are considered: geometrical nonlinearity only and geometric nonlinearity as well as material nonlinearity. Initial imperfection value is 1/1000. The analysis results are shown in Table 1, where the parameter A is considered only for the load characteristic values of the geometric nonlinearity, the B represents the load characteristic values of the double nonlinearity, and the ratio \( \lambda \) equals (a-b)/a.

<table>
<thead>
<tr>
<th>Order</th>
<th>A (geometric nonlinearity)</th>
<th>B (double nonlinearity)</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15.1976</td>
<td>8.412248</td>
<td>0.446</td>
</tr>
<tr>
<td>2</td>
<td>15.1637</td>
<td>8.466914</td>
<td>0.442</td>
</tr>
<tr>
<td>3</td>
<td>15.2392</td>
<td>7.883963</td>
<td>0.483</td>
</tr>
<tr>
<td>4</td>
<td>18.9454</td>
<td>10.80373</td>
<td>0.430</td>
</tr>
<tr>
<td>5</td>
<td>18.9487</td>
<td>9.940423</td>
<td>0.475</td>
</tr>
<tr>
<td>6</td>
<td>15.7261</td>
<td>8.504236</td>
<td>0.459</td>
</tr>
<tr>
<td>7</td>
<td>15.6682</td>
<td>8.179096</td>
<td>0.478</td>
</tr>
<tr>
<td>8</td>
<td>15.5924</td>
<td>7.393743</td>
<td>0.526</td>
</tr>
<tr>
<td>9</td>
<td>16.8673</td>
<td>9.49423</td>
<td>0.437</td>
</tr>
<tr>
<td>10</td>
<td>15.6605</td>
<td>8.589202</td>
<td>0.452</td>
</tr>
<tr>
<td>11</td>
<td>15.6352</td>
<td>8.632925</td>
<td>0.448</td>
</tr>
<tr>
<td>12</td>
<td>16.4821</td>
<td>8.799734</td>
<td>0.466</td>
</tr>
<tr>
<td>13</td>
<td>16.3482</td>
<td>9.168697</td>
<td>0.439</td>
</tr>
<tr>
<td>14</td>
<td>16.3678</td>
<td>9.372969</td>
<td>0.427</td>
</tr>
<tr>
<td>15</td>
<td>16.3005</td>
<td>8.9126</td>
<td>0.453</td>
</tr>
</tbody>
</table>

### 3.2 Analysis of different Initial defects

Consider the impact of different initial imperfection on the nonlinear analysis results, the analysis results are listed in Table 2. Among them, the parameter A is expressed only by the geometric nonlinearity of the load...
characteristic values, and the parameter B is expressed by considering the double nonlinear characteristic values of the load. The ratio $\lambda$ equals $(a-b)/a.$

<table>
<thead>
<tr>
<th>Initial defects</th>
<th>A (geometric nonlinearity)</th>
<th>B (double nonlinearity)</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/300</td>
<td>13.2882</td>
<td>7.350917</td>
<td>0.447</td>
</tr>
<tr>
<td>1/400</td>
<td>12.7072</td>
<td>7.538233</td>
<td>0.407</td>
</tr>
<tr>
<td>1/500</td>
<td>12.3762</td>
<td>6.814171</td>
<td>0.449</td>
</tr>
<tr>
<td>1/600</td>
<td>12.2669</td>
<td>6.736681</td>
<td>0.451</td>
</tr>
<tr>
<td>1/700</td>
<td>13.1034</td>
<td>7.397462</td>
<td>0.435</td>
</tr>
<tr>
<td>1/800</td>
<td>13.5708</td>
<td>7.326414</td>
<td>0.460</td>
</tr>
<tr>
<td>1/900</td>
<td>14.5769</td>
<td>8.170769</td>
<td>0.439</td>
</tr>
<tr>
<td>1/1000</td>
<td>15.1976</td>
<td>7.984934</td>
<td>0.475</td>
</tr>
<tr>
<td>1/1100</td>
<td>15.7597</td>
<td>9.2793</td>
<td>0.410</td>
</tr>
<tr>
<td>1/1200</td>
<td>16.2699</td>
<td>8.464467</td>
<td>0.480</td>
</tr>
<tr>
<td>1/1300</td>
<td>16.683</td>
<td>9.793517</td>
<td>0.413</td>
</tr>
<tr>
<td>1/1400</td>
<td>17.1523</td>
<td>9.631059</td>
<td>0.438</td>
</tr>
<tr>
<td>1/1500</td>
<td>17.5397</td>
<td>9.730659</td>
<td>0.445</td>
</tr>
</tbody>
</table>

The load factor is determined by the vertices of the graphs below.

Fig. 5 The load displacement curves of the geometric nonlinearity of the 1/300 initial imperfections

Fig. 6 The load displacement curves of the geometric nonlinearity of 1/500 initial imperfection
4. Conclusions

Conclusions can be reached from the analysis above:

(1) Considering that the load factor of the dual nonlinearity is less than the load factor of the geometric nonlinearity, the geometric nonlinearity is considered to be not satisfied with the safety requirements.

(2) The minimum load factor obtained by nonlinear analysis is not necessarily in the first order buckling mode, so what the Specification recommended that using the factor of the first order buckling mode as the nonlinear analysis result is unreasonable.

(3) The smaller the initial defect value comes the greater the value of the buckling load.

Acknowledgement

Project: Project of Science and Technology of Guangdong Province Transportation Department (Technology-2013-02-089)

References


Fig.7 The load displacement curves of the dual nonlinearity of 1/500 initial imperfection